Hydro-Cascade Model and Relativistic Kinetic Equations for Finite Domains

Kyrill A. Bugaev
Nuclear Science Division, Lawrence Berkeley National Laboratory, Berkeley, California 94720
Bogolyubov Institute for Theoretical Physics, Kiev, Ukraine

The modern history of relativistic hydrodynamics started more than fifty years ago when L. D. Landau suggested its use to describe the expansion of the strongly interacting matter that is formed in high energy hadronic collisions. Since that time there arose a fundamental problem of relativistic hydrodynamics known as the freeze-out problem. In other words, one has to know how to stop solving the hydrodynamical equations and convert the matter into free streaming particles. Several ways have been suggested to handle it, but only recently a new approach to solve the freeze-out problem in relativistic hydrodynamics has been invented by Bass and Dumitru (BD model) [1] and further developed by Teaney, Lauret and Shuryak (TLS model) [2]. These hydro + cascade models assume that the nucleus-nucleus collisions proceed in three stages: hydrodynamic expansion (hydro) of the quark gluon plasma (QGP), phase transition from the QGP to the hadron gas (HG) and hadronic rescattering and resonance decays (cascade). The switch from hydro to cascade modeling takes place at the boundary between the mixed and hadronic phases. The spectrum of hadrons leaving this boundary of the QGP–HG transition is taken as input for the cascade.

This approach incorporates the best features of both the hydrodynamical and cascade descriptions. It allows for, on one hand, the calculation of the phase transition between the quark gluon plasma and hadron gas using hydrodynamics and, on the other hand, the freeze-out of hadron spectra using the cascade description. This approach allows one to overcome the usual difficulty of transport approach in modeling phase transition phenomenon. However, both the BD and TLS models face some fundamental difficulties which cannot be ignored (see a detailed discussion in [3]). The BD approach leads to the causal paradox, whereas the TLS model does not conserve energy, momentum and number of charges due to the fact that the equations of motion used in [2] are incomplete and, hence, should be modified.

To overcome these difficulties the system of two coupled kinetic equations

$$\Theta_A\, p^\mu\, \partial_\mu\, \phi_A(x,p) = C^I_A(x,p) + C^{II}_A(x,p) + p^\mu \partial_\mu \mathcal{F}^* \times \\ [\phi_{in}(x,p) - \phi_{out}(x,p)]\; \Theta(S_A\; p^\nu \partial_\nu \mathcal{F}^*)\; \delta(\mathcal{F}^*(t,\bar{x})) \qquad (1)$$

was derived in [4] for the distributions ϕ_A $(A \in \{in; out\})$ of two domains "in" and "out", separated by the hypersurface Σ^* (see Fig. 1). This system explicitly accounts for the exchange of particles on the time-like parts of Σ^* (the δ-term in the r.h.s. of (1) is infinite at the FO arc in Fig. 1) and the switch off condition $\mathcal{F}^*(t,\bar{x}) = 0$. Another important feature of the equations (1) is that each of these equations identically vanishes outside of the domain of its existence due to the properties of Θ_A- and δ-functions. The system (1) not only generates the correct macroscopic conservation laws [3] on the hydro domain ("in"), but also it automatically generates the usual Boltzmann equation [4] for the full distribution function $\Phi(x,p) \equiv \Theta_{in}\, \phi_{in}(x,p) + \Theta_{out}\, \phi_{out}(x,p)$ without *any assumption* about the behavior of ϕ_{in} and ϕ_{out} on the boundary hypersurface Σ^*.

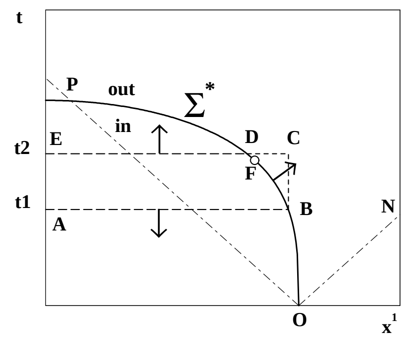

FIG. 1: Schematic two dimensional picture of the boundary hypersurface Σ^* (solid curve). Arrows show the external normal vectors. The light cone NOP is shown by the dash-dotted line. The point F divides Σ^* into the time-like (OF) and space-like (FP) parts.

The system (1) can be generalized to many particle species and the obtained macroscopic conservations laws can be solved at the boundary Σ^* between the domains. The developed formalism not only reproduces all kinds of discontinuities (usual shocks, time-like shocks, freeze-out shocks) known in relativistic hydro, but also generates the new, most general, kind of shock which I named a *three flux discontinuity* because it involves three irreducible fluxes. It is shown that a *three flux discontinuity* provides the correct initial condition for the cascade and simultaneously it obeys all conservation laws on Σ^* and does not lead to any causal paradox. Therefore, application of these results to the actual hydro+cascade simulations may open a principally new possibility not only to resolve HBT puzzles [5], but also to study some new phenomena, like a turbulence pattern, associated with a *three flux discontinuity* in relativistic hydro+cascade approach.

[1] S. Bass and A. Dumitru, Phys. Rev.**C 61**, 064909 (2000);
[2] D. Teaney, J. Lauret and E. V. Shuryak, Phys. Rev. Lett. **86**, 4783 (2001); and arXiv:nucl-th/0110037.
[3] K. A. Bugaev, Phys. Rev. Lett. **90**, 252301 (2003).
[4] K. A. Bugaev, arXiv:nucl-th/0401060.
[5] M. Gyulassy, Lect. Notes Phys. **583**, 37 (2002).